Quantum pump effect in one-dimensional systems of Dirac fermions

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We investigate the behavior of the directed current in one-dimensional systems of Dirac fermions driven by local periodic potentials in the forward as well in backscattering channels. We treat the problem with Keldysh non-equilibrium Green's function formalism. We present the exact solution for the case of an infinite wire and show that in this case the dc current vanishes identically. We also investigate a confined system consistent in an annular arrangement coupled to a particle reservoir. We present a perturbative treatment that allows for the analytical expressions of the dc current in the lowest order of the amplitudes of the potential. We also present results obtained from the exact numerical solution of the problem.

I. INTRODUCTION

The possibility of employing ac-fields to induce directed flow of charge and spin in quantum systems is in the limelight of quantum transport phenomena. Recent experimental observations of the quantum pumping effect, including the periodic deformation of the walls of quantum dots [1, 2, 3, 4] and the induction of currents by applying surface acoustic waves in carbon nanotubes [5] triggered an important development in the theoretical description of time-dependent quantum transport [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

The development of new technologies based on quantum phenomena, is also gauging the search of novel materials to achieve transport in devices of reduced scale. Among the many structures under investigation, devices fabricated of graphite are capturing an increasing attention. From the theoretical side, the Physics of graphite has the appealing feature of being a realization of a gas of Dirac fermions in reduced dimensions. In fact, the behavior of the conductance of single-wall nanotubes [23] seems to be reasonably explained on the basis of a Lüttinger liquid, which is one of the few models of interacting electrons in condensed matter Physics that is amenable to be analytically solved. The Physics of graphene, being a single graphite sheet seems also to be suitably described as a two-dimensional gas of Dirac fermions [24]. These developments are motivating a renewed interest in the Physics of Dirac fermions and in the application of quantum field theoretical methods to shed light on low dimensional Condensed Matter problems. In particular, a precise understanding of the effect of time-dependent perturbations on these systems will be crucial in order to build nanodevices based on the pumping of charge and spin into a conductor. Recently, we have solved the problem of an ac-potential in a one-dimension infinite wire of Dirac fermions, emphasizing the separate role of the forward and backward channels in the resulting total energy density and transport properties [25].

The breaking of certain symmetries has been singled out as a necessary condition to generate directed currents. This idea has been firstly introduced in the context of classical "ratchet" systems, [26] and also seems to apply in paradigmatic models of quantum pumps, like two barrier structures driven by two harmonic fields oscillating with a phase-lag. [21] We recall that the pump effect is characterized by the induction of a directed current with pure ac driving, i.e. without introducing any explicit dc bias voltage. In this work we adapt the two oscillating barriers setup to a one-dimensional wire of Dirac fermions. One of our aims is to identify which are the minimal ingredients to achieve the quantum pump effect in these systems. An infinite wire of Dirac fermions with two local forward ac fields oscillating with a phase lag can be analytically solved by generalizing the treatment of Ref. 25 As we will show, the ensuing induced current has a vanishing dc component. Then, it is natural to explore the role of confinement in relation to the appearance of non vanishing dc currents. This is the main purpose of this work. In particular, we consider a finite wire of length L bent into a ring and coupled to an external reservoir while driven by ac forward and backward fields, as indicated in the sketch of Fig. 1. This setup enables us to analyze the role of dynamical forward and backscattering in wires containing a spectrum with many resonant levels, being in contact with a dissipative environment. We treat the problem with Keldysh non-equilibrium Green's functions [27] and find analytical expressions for the dc current at the lowest order of perturbation theory in the pumping amplitudes. We also present a system of exact equations and exact results for the dc current obtained by the numerical solution of these equations.

The paper is organized as follows. In sections II and III
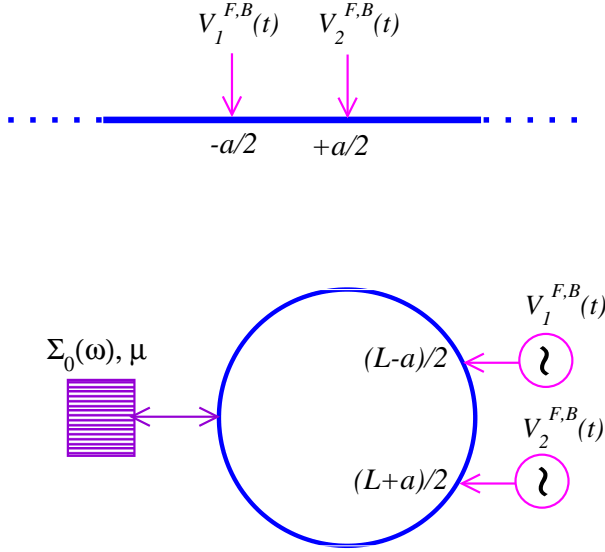
FIG. 1: (Color online) Sketch of the setup. Above: Infinite wire driven by periodic fields inducing dynamic forward and back scattering at the positions $x = \pm a/2$. Bellow: The ac fields act on a finite wire of length L in anular geometry. The system is coupled to a particle reservoir a the position $x = 0$.

we present the model and the theoretical treatment. In section IV we present the exact evaluation of the dc current in an infinite wire with dynamical fowrd scattering. Section V is devoted to study the transport behavior of a ring with finite length coupled to a reservoir. A summary and conclusions are presented in Section VI. Technical details, in particular, the procedure followed to obtain an effective action through the functional integration of the reservoir's fields, are described in Appendix A.

II.   MODELS

We study a one-dimensional system of Dirac fermions in two different setups. Firstly, we consider an infinite wire with two harmonically time-dependent local potentials applied at the positions $x_j = \pm a/2$ (see Fig. 1a). This system is described by an action

$$S = S_0 + S_{ac}, \qquad (1)$$

where S_0 is the linearized free dispersion relation for right and left-movers (in our unit system $\hbar = v_F = e = 1$):

$$S_0 = i \int dx\, dt\, (\psi_r^\dagger(t)(\partial_t + \partial_x)\psi_r(t) + \psi_l^\dagger(t)(\partial_t - \partial_x)\psi_l(t)), \qquad (2)$$

S_{ac} describes forward and backward scattering processes through the two time-dependent potentials:

$$\begin{aligned}
S_{ac} = \sum_{j=1}^{2} \int dx\, dt\, \delta(x - x_j) \Big[& V_j^F(t)(\psi_r^\dagger(t)\psi_r(t) + \\
& +\psi_l^\dagger(t)\psi_l(t)) + V_j^B(t)(\psi_r^\dagger(t)\psi_l(t) + \\
& +\psi_l^\dagger(t)\psi_r(t)) \Big].
\end{aligned} \qquad (3)$$

Note that, for simplicity, the dependence of the fields on x has been omitted in the above equations. To be specific we consider harmonic potentials with equal amplitudes and a phase-lag δ: $V_1^{F,B}(t) = V^{F,B}\cos\Omega_0 t$ and $V_2^{F,B}(t) = V^{F,B}\cos(\Omega_0 t + \delta)$, where F, B denote forward and backward channels, respectively.

In the second configuration, we assume that the Dirac fermions propagate in a ring of length L, which is in contact to a fermionic reservoir. Also in this case, harmonically time-dependent local potentials are applied at the positions $x_i = (L \pm a)/2$. The action for the full system can be casted in terms of four terms:

$$S = S_0 + S_{ac} + S_{res} + S_{cont}. \qquad (4)$$

The reservoir is assumed to be an infinite system of free fermions which couples to electrons in the ring through a forward scattering term. The term S_{cont} account for the interaction of fermions with the reservoir, placed at $x = 0$. It reads:

$$\begin{aligned}
S_{cont} = \int dx\, dt\, v_c\, \delta(x)(& \psi_r^\dagger(t)\chi_r(t) + \psi_l^\dagger(t)\chi_l(t) + \\
& +\chi_r^\dagger(t)\psi_r(t) + \chi_l^\dagger(t)\psi_l(t)),
\end{aligned} \qquad (5)$$

where the fields $\chi_r^\dagger(t), \chi_l^\dagger(t)$ denote degrees of freedom of the reservoir. The latter can be integrated out, giving rise to an effective self-energy $\Sigma_0(t - t')$ (see Appendix A) which leads to the following effective action for the internal degrees of freedom of the ring:

$$\begin{aligned}
S_{eff} = \int dx\, dt\, dt' \Big(& \psi_r^\dagger(t) \\
& \Big[i(\partial_t + \partial_x)\delta(t - t') - \delta(x)\,\Sigma_0(t - t')\Big]\psi_r(t') \\
& +\psi_l^\dagger(t)\Big[i(\partial_t - \partial_x)\delta(t - t') - \delta(x)\,\Sigma_0(t - t')\Big]\psi_l(t') \Big) \\
& +\sum_{i=1}^{2} \int dx\, dt\, \delta(x - x_i) \\
& \times \Big[V_i^F(t)(\psi_r^\dagger(t)\psi_r(t) + \psi_l^\dagger(t)\psi_l(t)) \\
& +V_i^B(t)(\psi_r^\dagger(t)\psi_l(t) + \psi_l^\dagger(t)\psi_r(t)) \Big].
\end{aligned} \qquad (6)$$

Our main purpose is to compute the dc component of the time-dependent charge current. The latter is defined through the condition of the charge conservation:

$$\frac{d\rho_l(x,t)}{dt} + \frac{d\rho_r(x,t)}{dt} = \frac{\partial j(x,t)}{\partial x}, \qquad (7)$$

being $\rho_\gamma(x,t) = \langle \psi_\gamma^\dagger(x,t)\psi_\gamma(x,t)\rangle$, $\gamma = l, r$, which casts:

$$j(x,t) = \rho_r(x,t) - \rho_l(x,t). \quad (8)$$

The corresponding dc current is

$$j^{dc} = \frac{1}{\tau_0}\int_0^{\tau_0} dt\, j(x,t), \quad (9)$$

where $\tau_0 = 2\pi/\Omega_0$ is the period of the oscillations. Note that, due to the conservation of the charge, (see eq. (7)), it does not depend on the spacial coordinate x.

III. THEORETICAL TREATMENT

We solve the problem presented in the previous section by recourse to Keldysh non equilibrium Green's functions. This formalism is based on the solution of the equation of motion of the matricial Green's function obtained from the corresponding action.

From actions (2) and (3) it is easy to show that the matricial Green's function $\mathcal{G}$ reads:

$$\mathcal{G}_{\gamma,\gamma'}(x,x',t,t') = \begin{pmatrix} G_{\gamma,\gamma'}^{++}(x,x',t,t') & G_{\gamma,\gamma'}^{+-}(x,x',t,t') \\ G_{\gamma,\gamma'}^{-+}(x,x',t,t') & G_{\gamma,\gamma'}^{--}(x,x',t,t') \end{pmatrix} \quad (10)$$

where the indexes $+, -$ refer to the closed time path of Keldysh formalism [27], being

$$\begin{aligned} G_{\gamma,\gamma'}^{++}(x,x',t,t') &= -i\langle T\psi_\gamma(x,t)\psi_{\gamma'}^\dagger(x',t')\rangle, \\ G_{\gamma,\gamma'}^{--}(x,x',t,t') &= -i\langle \tilde{T}\psi_\gamma(x,t)\psi_{\gamma'}^\dagger(x',t')\rangle, \\ G_{\gamma,\gamma'}^{+-}(x,x',t,t') &= i\langle \psi_{\gamma'}^\dagger(x',t')\psi_\gamma(x,t)\rangle, \\ G_{\gamma,\gamma'}^{-+}(x,x',t,t') &= -i\langle \psi_\gamma(x,t)\psi_{\gamma'}^\dagger(x',t')\rangle, \quad (11) \end{aligned}$$

where T and $\tilde{T}$ denote temporal and anti-temporal ordering, respectively. These functions satisfy the following Dyson equation:

$$\begin{aligned} \Big([i(\partial_t &+ s_\gamma\partial_x) - V^F(x,t)]\delta_{\gamma,\gamma_1} - \\ &- V^B(x,t)\delta_{\gamma,-\gamma_1}\Big)\mathcal{G}_{\gamma_1,\gamma'}(x,x',t,t') \\ &- \delta(x)\delta_{\gamma,\gamma_1}\int dt_1 \Sigma_0(x,t-t_1)\mathcal{G}_{\gamma_1,\gamma'}(x,x',t_1,t') \\ &= \delta_{\gamma,\gamma'}\delta(x-x')\,\delta_C(t-t') \quad (12) \end{aligned}$$

where $\delta_C(t-t')$ is the matricial delta function (extended to the time contour):

$$\delta_C(t-t') = \begin{cases} \delta(t-t') & t,t' \text{ on } C_+ \\ -\delta(t-t') & t,t' \text{ on } C_- \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

$V^{F,B}(x,t) = \delta(x-x_1)V_1^{F,B}(t) + \delta(x-x_2)V_2^{F,B}(t)$, $s_\gamma = +,-$ for the right and left movers, respectively, and

$$\Sigma_0(t-t') = \begin{pmatrix} \Sigma_0^{++}(t-t') & \Sigma_0^{+-}(t-t') \\ \Sigma_0^{-+}(t-t') & \Sigma_0^{--}(t-t') \end{pmatrix}. \quad (14)$$

In terms of Green's functions, the current (8) reads

$$j(x,t) = -i[G_{rr}^{+-}(x,x,t,t) - G_{ll}^{+-}(x,x,t,t)]. \quad (15)$$

As shown in the next section, for the infinite system with $\Sigma_0(t-t') = 0$, the Green's function admits a factorization in terms of the noninteracting (equilibrium) Green's function, at least for ac potentials that behave as forward scatterers. One then obtains an analytical expression for $j(x,t)$ with vanishing time average. This result can be extended perturbatively to the backscattering case.

The situation changes dramatically when fermions propagate in a ring connected to a reservoir. As a consequence of a non-instantaneous term associated to the coupling to the reservoir, the equation of motion for the Green's function ceases to be exactly solvable even for the forward channel and its solution must be found by recourse to perturbation theory or numerical methods.

IV. INFINITE WIRE

For the infinite wire, we can adapt the treatment of Ref. [25], to the case of more than one oscillating potentials. For the case of pure forward ac potentials (i.e. $V_i^B = 0, \forall i$), $j(x,t)$ can be exactly evaluated and the result is a purely ac current. To show this we solve the Dyson equation (12) for $\Sigma_0(t-t') = 0$ and two oscillating barriers placed at $x = \pm a/2$. Specializing eq. (12) to the case $V^B = 0$ and considering first, for illustrative purposes the Green's function for right-movers, results

$$(i(\partial_t + \partial_x) - V^F(x,t))\mathcal{G}(x,x',t,t') = \delta(x-x')\,\delta_C(t-t') \quad (16)$$

It is very useful to note that the solution of the above differential equation for the matrix $\mathcal{G}$ can be obtained in terms of the free function $\mathcal{G}^0$, the latter being the solution of (16), with $V^F(x,t) = 0$. The result is:

$$\mathcal{G}(x,t;x',t') = \mathcal{G}^0(x-x',t-t')\exp[\beta(x,t) - \beta(x',t')], \quad (17)$$

with

$$\beta(x,t) = \int dx'\,dt'\, G^{0,R}(x-x',t-t')V^F(x',t'), \quad (18)$$

where $G^R(x-x',t-t') = G^{0,++} - G^{0,+-}$ is the equilibrium retarded Green's function of the free system. The above equation can be readily solved to obtain:

$$\begin{aligned} \beta(x,t) = -iV^F\Big(&\theta(x-a/2)\cos\Omega_0(t-(x-a/2)) + \\ &+ \theta(x+a/2)\cos\left(\Omega_0(t-(x+a/2)) + \delta\right)\Big) \quad (19) \end{aligned}$$

Having now $\beta(x,t)$ we have, via equation (17), every component of $\mathcal{G}$. Of course, the same steps can be followed to evaluate the Green's functions for left-movers. The functions $G_{\gamma,\gamma}^{+-}(x,x',t,t')$ can be used to compute the current

from eq. (15). This leads to

$$
\begin{aligned}
j(x,t) &= \frac{\Omega_0 V^F}{2\pi}\Big(\theta(x+a/2)\sin[\Omega_0(t-(x+a/2))] \\
&+ \theta(x-a/2)\sin[\Omega_0(t-(x-a/2))+\delta] \\
&+ \theta(-(x+a/2))\sin[\Omega_0(t+x+a/2)] \\
&+ \theta(-(x-a/2))\sin[\Omega_0(t+x-a/2)+\delta]\Big) (20)
\end{aligned}
$$

which yields $j_{wire}^{dc}=0$.

In the case $V^B \neq 0$ it is not possible to arrive at a closed expression for the current, one is forced to perform a perturbative expansion. We have computed $j(x,t)$ up to second order in V^B, showing that even for this case the time average of $j(x,t)$ vanishes for the infinite wire.

V. FINITE RING IN CONTACT TO A RESERVOIR

A. Evaluation of the Green's function

In the case of the annular setup, the presence of a self-energy in the effective action S_{eff} renders the procedure of the previous section unsuitable even in the case of pure forward dynamical scattering. However, the Green's functions can be formally exactly calculated by following a similar route to the one adopted in the solution of time-dependent lattice Hamiltonians [20]. In what follows we summarize this procedure. It is convenient to combine the different components of the Greens function matrix to define a retarded Green's function:

$$
\begin{aligned}
G_{\gamma,\gamma'}^R(x,x',t,t') &= \Theta(t-t')[G_{\gamma,\gamma'}^{-+}(x,x',t,t') \\
&- G_{\gamma,\gamma'}^{+-}(x,x',t,t'), \quad (21)
\end{aligned}
$$

and to perform a Fourier transform with respect to the difference of times:

$$
G_{\gamma,\gamma'}^R(x,x',t,t') = \int \frac{d\omega}{2\pi} G_{\gamma,\gamma'}^R(x,x',t,\omega)\, e^{-i\omega(t-t')}. \quad (22)
$$

Substituting it in the integral representation of the Dyson's equations (12), the following set of linear equations is obtained [20]:

$$
\begin{aligned}
G_{\gamma,\gamma'}^R(x,x',t,\omega) &= G_\gamma^0(x,x',\omega) \\
&+ \sum_{j=1}^2 e^{-i(\Omega_0 t-\delta_j)} G_{\gamma,\gamma'}^R(x,x_j,t,\omega+\Omega_0) \\
&\times \Big[V^F\delta_{\gamma,\gamma'} + V^B\delta_{\overline{\gamma},\gamma'}\Big] G_{\gamma'}^0(x_j,x',\omega) \\
&+ \sum_{j=1}^2 e^{(i\Omega_0 t+\delta_j)} G_{\gamma,\gamma'}^R(x,x_j,t,\omega-\Omega_0) \\
&\times \Big[V^F\delta_{\gamma,\gamma'} + V^B\delta_{\overline{\gamma},\gamma'}\Big] G_{\gamma'}^0(x_j,x',\omega), \quad (23)
\end{aligned}
$$

being $\overline{r}=l$ and $\overline{l}=r$. The function $G_\gamma^0(x_j,x',\omega)$ is the retarded Green's function of Dirac left and right movers

of the ring in contact to the reservoir with $V_i(t)=0$. This function can be calculated by considering the corresponding Dyson equation, which in the present case takes the following simple form:

$$
\begin{aligned}
G_\gamma^0(x,x',\omega) &= g_\gamma^0(x,x',\omega) \\
&+ G_\gamma^0(x,0,\omega)\Sigma_0^R(\omega)g_\gamma^0(0,x',\omega), \quad (24)
\end{aligned}
$$

where $g_\gamma^0(x,x',\omega)$ are the free retarded functions, corresponding to the ring without driving and uncoupled from the reservoir, while $\Sigma_0^R(\omega)$ is the Fourier transform of the retarded self-energy: $\Sigma_0^R(t-t')=\Theta(t-t')[\Sigma_0^{-+}(t-t')-\Sigma_0^{+-}(t-t')]$. It can be in general expressed in terms of its spectral representation $\Gamma(\omega)=-2\text{Im}[\Sigma_0^R(\omega)]$ as follows:

$$
\Sigma_0^R(\omega) = \int_{-\infty}^\infty \frac{d\omega}{2\pi}\frac{\Gamma(\omega)}{\omega-\omega+i0^+}. \quad (25)
$$

The equation for the lesser function reads:

$$
\begin{aligned}
G_{\gamma,\gamma'}^{+-}(x,x',t,t') &= \sum_{\gamma_1=l,r}\int_{-\infty}^\infty \frac{d\omega}{2\pi} e^{-i\omega(t-t')}G_{\gamma,\gamma_1}^R(x,0,t,\omega) \\
&\times \Sigma_0^{+-}(\omega)[G_{\gamma',\gamma_1}^R(0,x',t',\omega)]^*. \quad (26)
\end{aligned}
$$

This function can be used to evaluate current from (15). Recalling that $\Sigma_0^{+-}(\omega)=if(\omega)\Gamma(\omega)$, where $f(\omega)=\theta(-(\omega-\mu))$ is the Fermi function (we shall consider the zero-temperature case, μ is the chemical potential of the reservoir), the dc current can, thus, be expressed as follows:

$$
j^{dc} = \int \frac{d\omega}{2\pi} f(\omega)T(\omega), \quad (27)
$$

where the functions $T(\omega)$ collects the spectral contributions to the current from the forward and backward channels. Concretely:

$$
\begin{aligned}
T(\omega) &= \frac{1}{\tau_0}\int_0^{\tau_0} dt\Big(\mid G_{rr}^R(x,0,t,\omega)\mid^2 \\
&- \mid G_{ll}^R(x,0,t,\omega)\mid^2 + \mid G_{rl}^R(x,0,t,\omega)\mid^2 \\
&- \mid G_{lr}^R(x,0,t,\omega)\mid^2\Big)\Gamma(\omega), \quad (28)
\end{aligned}
$$

Therefore, the evaluation of the retarded Greens functions leads us to the current. To this end, we must solve Dyson's equation (23).

B. Perturbative expansion in the pumping amplitudes

In view of the obvious difficulties of solving equation (23), we propose in this section a perturbative solution that allows us to evaluate j^{dc} to order $\mathcal{O}(|V^F|^2,|V^B|^2)$. We will obtain an analytical expression for j^{dc}, through the "transmission function" $T(\omega)$, valid for any kind of fermionic reservoir interacting via forward scattering

with the fermions in the ring, like the one described by S_{cont} introduced in eq. (5).

Before going to the perturbative treatment, let us note that given the Green's function of the free system:

$$g_r^0(x, x', \omega) = g_l^0(x', x, \omega) = \lim_{\Lambda \to \infty} \frac{\pi}{\Lambda} \frac{\exp i\omega(x - x' - L/2)}{2 \sin \omega L/2},$$
(29)

Eq. (24) can be exactly solved:

$$G_r^0(x, x', \omega) = G_l^0(x', x, \omega) = \lim_{\Lambda \to \infty} \frac{\pi}{\Lambda} \frac{\exp i\omega(x - x' - L/2)}{h(\omega)},$$
(30)

with

$$h(\omega) = 2 \sin \omega L/2 - (\Sigma_0^R(\omega))e^{-i\omega L/2}.$$
(31)

The normalization factor π/Λ is introduced because the functions $G_\gamma^0(x, x, \omega)$ and $g_\gamma^0(x, x', \omega)$ are not regular, in the sense that the densities of states $\rho_\gamma(x, x, \omega) = -2\text{Im}[G_\gamma^0(x, x, \omega)]$ and $\rho_\gamma^0(x, x, \omega) = -2\text{Im}[g_\gamma^0(x, x, \omega)]$ do not integrate to 1. Actually, the ensuing integrals diverge and, for this reason, upper and lower cutoffs $\pm\Lambda$ are introduced to ensure the correct sum rule. In the case of the Green's function for the ring coupled to the reservoir, the normalization is the same as for the free system provided that the reservoir has a finite bandwidth W, such that $\Gamma(w) = 0$ for $|\omega| > W$, and $\Lambda >> W$.

Given a concrete model for the reservoir, which implies assuming a concrete functional form for the spectral density $\Gamma(\omega)$, one has a definite function $h(\omega)$ (See (31)). This, in turn, leads to the retarded functions for the system without time-dependent potentials, given in (24). These functions are our building blocks for the perturbative solution of the full retarded Greens function $G_{\gamma\gamma}^R(x, x', t, \omega)$. Solving equation (23) up to second order in the couplings and replacing the results in (28), we get

$$T(\omega) = \frac{-\pi^2}{\Lambda^2} \frac{\Gamma(\omega)}{|h(\omega)|^2} \sin \delta \times$$
$$\left(V_F^2 \sin \Omega_0 a \left[T^{(1)}(\omega) + T^{(2)}(\omega) \right] - \right.$$
$$\left. -V_B^2 [T^{(3)}(\omega) + T^{(4)}(\omega)] \right),$$
(32)

where

$$T^{(1)}(\omega) = \frac{1}{|h(\omega + \Omega_0)|^2} + \frac{1}{|h(\omega - \Omega_0)|^2},$$
(33)

$$T^{(2)}(\omega) = 2Re\frac{e^{i\omega L}}{h^*(\omega)} \left(\frac{e^{i\Omega_0 L/2}}{h^*(\omega + \Omega_0)} + \frac{e^{-i\Omega_0 L/2}}{h^*(\omega - \Omega_0)} \right),$$
(34)

$$T^{(3)}(\omega) = 2Re\frac{e^{i\omega L}}{h^*(\omega)} \left(\frac{\sin((2\omega - \Omega_0)a)e^{i\Omega_0 L/2}}{h^*(\omega + \Omega_0)} - \right.$$
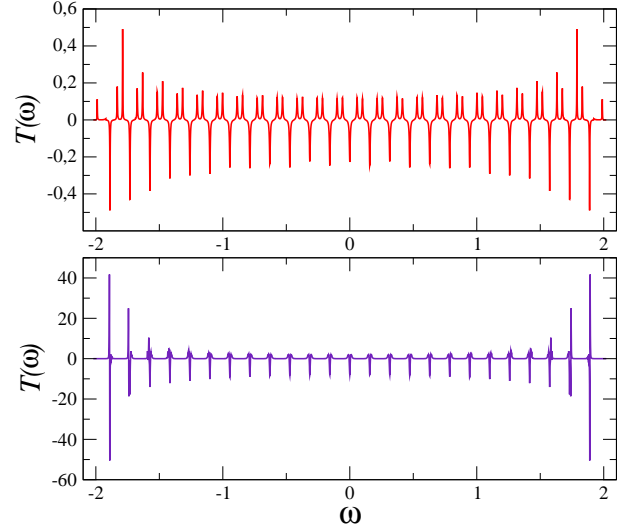$$\left. \frac{\sin((2\omega + \Omega_0)a)e^{-i\Omega_0 L/2}}{h^*(\omega - \Omega_0)} \right)$$

FIG. 2: (Color online) The function $T(\omega)$ for pure forward scattering. Parameters are: $V_F = 0.2$, $\delta = \pi/2$, $a = 2$, $L = 40$. We consider a reservoir with bandwidth $W = 16$ and a coupling constant $v_c = 0.1$. Upper and lower panels correspond to $\Omega_0 = 0.1, 0.15$, respectively.

$$T^{(4)}(\omega) = \frac{\sin(2\omega - \Omega_0)a}{|h(\omega - \Omega_0)|^2} - \frac{\sin(2\omega + \Omega_0)a}{|h(\omega + \Omega_0)|^2}.$$
(35)

In equation (32) we have omitted, for sake of simplicity, the limit $\Lambda \to \infty$. Let us stress again that these formulas do not depend on the specific choice of the function $\Gamma(\omega)$. This allows us to obtain some general properties of quantum pumping in the annular geometry which are independent of the details of the reservoir. As a first point, we stress that unlike the case of the infinite wire, in the finite ring, forward scattering is enough to generate a directed current. Second, it is interesting to note that we recover the behavior $j^{dc} \propto \sin(\delta)$ predicted in Ref. [7] for a two-barrier quantum pump in a linear setup, also obtained within Floquet scattering matrix formalism [16] as well as within Green's function formalism in tight-binding models [20, 21]. As a third point, for $\delta \neq mod(\pi)$ and $\Omega_0 a \ll 1$, the dc current behaves like $j^{dc} \propto \Omega_0$, which is the typical behavior of the so called "adiabatic" regime of quantum pumps [7, 11, 16, 17, 18]. A final and important point to note is the role of geometrical factors, like the length L and the situation of the barriers a. In particular, the prefactor in (32) indicates that for weak amplitudes and special combinations of parameters such that $\Omega_0 a = \pi$, the dc current is inhibited in the forward channel. In addition, geometrical factors lead to sign modulation of the induced dc current as a function of the chemical potential μ, as we will discuss in more detail in the next section.
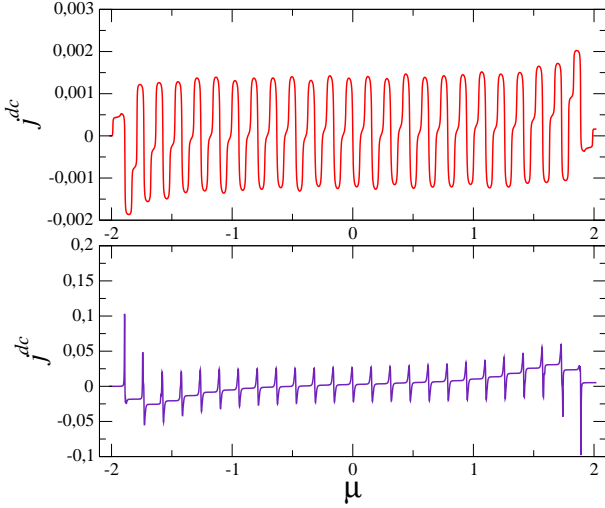
FIG. 3: (Color online) dc current J^{dc} as a function of the chemical potential of the reservoir for pure forward scattering. Parameters are the same as in Fig. 2.

FIG. 4: (Color online) The function $T(\omega)$ for pure backward scattering with $V_B = 0.2$. Other details are the same as in Fig. 3.

C. Exact numerical solution

In this section we present results obtained from the numerical exact solution of the Dyson's equation (23). In order to avoid the problem of the regularization of the Green's function discussed in the previous section, we consider in this section functions

$$g_\gamma^0(x, x', \omega) = \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{e^{-ik_n(x-x')}}{\omega \pm k_n + i0^+}, \qquad (36)$$

being $k_n = 2\pi/L$ while the upper (lower) sign corresponds to $\gamma = r, l$. This function coincides with (29) in the limit of $N \to \infty$, and has well defined spectral properties, i.e. it satisfies the usual sum rule for the spectral density. The drawback of this Green's function is that Dyson's equation (24) cannot be analytically solved. In any case, the equilibrium Green's functions can be numerically evaluated from the expression:

$$\begin{aligned} G_\gamma^0(x, x', \omega) &= g_\gamma^0(x, x', \omega) \\ &\quad + \frac{g_\gamma^0(x, 0, \omega) g_\gamma^0(0, x', \omega)}{1 - \Sigma_0^R(\omega) g_\gamma^0(0, 0, \omega)}, \end{aligned} \qquad (37)$$

and used as input to the numerical evaluation of (23).

So far, we have not introduced any specific model for the reservoir. In the numerical procedure we have represented the latter system in terms of a semicircular density of states of bandwidth W and a coupling v_c between ring and reservoir. The ensuing self-energy is defined from (25) with $\Gamma(\omega) = 4|v_c|^2 \Theta(W - |\omega|) \sqrt{W^2 - \omega^2}/W^2$.

Results for the function $T(\omega)$ in the case of pure forward scattering are shown in Fig. 2. This function displays a landscape of peaks and anti-peaks that are in accordance with the structure predicted by the perturbative solution. For $V_B = 0$ only the functions $T^{(1)}(\omega)$
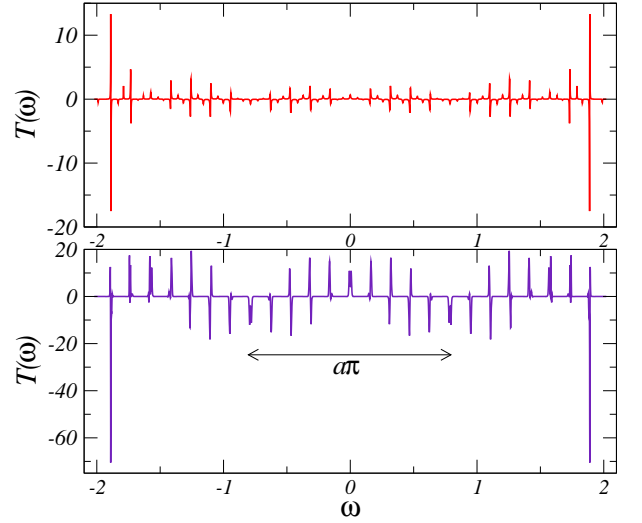
and $T^{(2)}(\omega)$ contribute (see Eqs. (32), (33) and (34)). Notice that these functions have peaks centered at the zeros of the function $h(\omega \pm \Omega_0)$ with $\Sigma_0(\omega) = 0$, which correspond to the positions of the energy levels of the uncoupled ring, i.e. at $\omega \pm \Omega_0 = 2n\pi/L$, with n integer. The modulation with period ωL of $T^{(2)}(\omega)$ also explains the pattern of alternative maxima and minima of $T(\omega)$ at the energy levels of the uncoupled ring. In the lower panel of Fig. 2 it is shown the behavior of $T(\omega)$ for $\Omega_0 \sim 2\pi/L$, i.e. close to resonance with the characteristic frequency associated to the level spacing of the uncoupled ring.

At zero temperature, the integral of $T(\omega)$ over energies below the chemical potential μ gives the dc component of current for a given μ. The behavior of J^{dc} as a function μ is shown in Fig. 3 for a driven system with the same parameters of 2. In the upper panel, corresponding to $\Omega_0 < 2\pi/L$, a sequence of plateaus of width Ω_0 are clearly distinguished. In addition sign inversion of the current takes place at points distant in the level spacing $2\pi/L$. In the case of the lower panel (close to resonance) the plateaus approximately coincide with the level spacing ($\sim \Omega_0$).

The function $T(\omega)$ for pure dynamical backscattering ($V_F = 0$) is shown in Fig. 4. The behavior of this function is also in agreement with the one predicted by the perturbative solution. In this case, only the functions $T^{(3)}(\omega)$ and $T^{(4)}(\omega)$ contribute. As in the case of pure forward scattering, these functions have a sequence of peaks and anti-peaks defined from $\omega \pm \Omega_0 = 2\pi/L$. However in the present case, the structure is richer than in the case of forward scattering. This is due to the existence of two modulating factors: $e^{i\omega L}$ in $T^{(3)}(\omega)$, and the $\sin[(2\omega \pm \Omega_0)a]$ in $T^{(3)}(\omega)$ as well as in $T^{(4)}(\omega)$. While the first factor introduces a periodicity which is coincident with the level spacing, the second one introduces a peri-
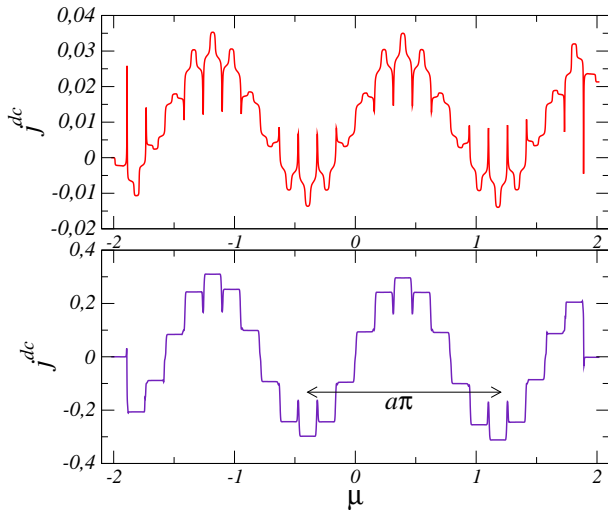
FIG. 5: (Color online) dc current J^{dc} as a function of the chemical potential of the reservoir for pure backward scattering. Other details are the same as in Fig. 4.

odicity πa that is defined by the position at which the ac potentials are applied. In addition, for backscattering, different terms contribute with different signs, while in the case of forward scattering all the contributions are additive.

In Fig. 4, an envelope with period $a\pi$ can be clearly distinguished and it is indicated with an arrow in the lower panel. The corresponding dc current is shown in Fig. 5. For low pumping frequencies (upper panel) as well as for frequencies close to resonance (lower panel), the behavior of the directed current as a function of the chemical potential displays sign inversions and a sequence of maxima and minima separated in $\pi a/2$. In addition, there is a fine structure of plateaus of width Ω_0, which is more pronounced in the case of low Ω_0 (upper panel). Also notice, and additional structure related to the level spacing energy $2\pi/L$ of the ring.

Thus, dynamical forward and backscattering mechanisms leave clearly different fingerprints in the behavior of the induced directed current as a function of the chemical potential of the reservoir. While both processes are sensitive and give rise to patterns related to the level spacing and the pumping frequency, backscattering leads to a response that also provides information on the positions at which the pumping potentials are applied. This feature accounts for interference processes in the electronic propagation that take place when electrons emit or absorb quanta and invert at the same time their direction of motion at the backscattering centers.

VI. SUMMARY AND CONCLUSIONS

In this work we have studied quantum pumping induced by two local ac potentials in the forward and backward channels, oscillating with a single frequency and a phase lag, in a one-dimensional system of Dirac fermions. This problem can be exactly solved analytically in the case of an infinite wire and pure forward scattering. The exact solution shows that the charge current in this case is purely ac without a dc component. For this reason, we have investigated a confined system of annular shape in contact to a particle reservoir. We have employed a formalism based on Keldysh non-equilibrium Green's functions to solve this problem. In this case, an analytical solution is available a the lowest order in the pumping amplitudes, while for arbitrary amplitudes, it can be exactly solved by numerically evaluating a Dyson equation for the retarded Green's functions.

We have shown that a directed current establishes in the confined geometry with dynamical forward as well as backscattering. As a function of the phase-lag δ, the current shows the $\propto \sin \delta$ behavior already presented in the literature in the framework of adiabatic descriptions [7], models of plane-wave [16] and tight-binding electrons, [20, 21] and experimentally observed in Ref. 1. Interestingly, in the case of pure forward scattering there is an additional factor $\propto \sin(\Omega_0 a)$, denoting interference effects related to the geometrical arrangement. As a function of the chemical potential of the reservoir, forward and backward scattering display features in the behavior of the dc current which are related to the wire level spacing the pumping frequency. In the case of backscattering, there is additional structure related to geometrical parameters.

VII. ACKNOWLEDGMENTS

APPENDIX A: INTEGRATION OF THE RESERVOIR'S DEGREES OF FREEDOM

In this appendix we show how to get the effective action given in eq. (6). To this end we consider the path-integral representation for the piece of the partition function that involves the fields χ_r and χ_l to be integrated:

$$\mathcal{Z}_{res} = \int d\chi_r^\dagger \, d\chi_l^\dagger \, d\chi_r \, d\chi_l \, \exp i(S_{res} + S_{cont}) \quad \text{(A1)}$$

where

$$S_{res} = \int dx \, dt \, (\chi_r^\dagger D_r \, \chi_r + \chi_l^\dagger D_r \, \chi_l) \quad \text{(A2)}$$

where D_r and D_l are differential operators which specific forms depend on the dispersion relations satisfied by the particles in the reservoir. Since the integration procedure can be performed quite generally, we will specify

the actual form for these operators at the end of the derivation. On the other hand, the interaction between the reservoir's modes and fermions in the ring is given by S_{cont} (see eq.(5)). In order to obtain an action which is quadratic in the reservoir's fields we perform a translation in these fields of the form $\chi_{r,l} \to \sigma_{r,l}$ such that:

$$\chi_{r,l}(x,t) = \sigma_{r,l}(x,t) + a_{r,l}(x,t), \qquad (A3)$$

(and similar transformations for $\chi_{r,l}^{\dagger}$). Choosing the functions $a_{r,l}$ as

$$a_{r,l}(x,t) = -v_c \int dx'\, dt'\, D_{r,l}^{-1}(x,x';t,t')\, \delta(x')\, \psi_{r,l}(x',t'), \qquad (A4)$$

(and similar expressions for $a_{r,l}^{\dagger}$) the resulting action is quadratic in the $\sigma_{r,l}$ fields, which are then integrated out.

Of course, there are also two terms depending quadratically on $\psi_{r,l}$ which will also contribute to the effective action. The general form of these action terms is:

$$- v_c^2 \int dx\, dt\, dt'\, \psi_{r,l}^{\dagger}(x,t)\, \delta(x)\, D_{r,l}^{-1}(0,0;t,t')\, \psi_{r,l}^{\dagger}(x,t). \qquad (A5)$$

In this paper we will consider particle reservoirs such that right and left propagators coincide for $x = x'$. We will also assume time translation invariance such that $D_{r,l}^{-1}(0,0;t,t') = D^{-1}(0,0;t-t')$. The reservoir's self-energy used in the paper is defined in terms of this propagator as $\Sigma_0(t-t') = v_c^2\, D^{-1}(0,0;t-t')$. An explicit form for this function is chosen in section IV C.

[1] M. Switkes, C. M. Marcus, K. Campman, A. C. Gossard, Science **283**, 1905 (1999).
[2] L. J. Geerligs, V. F. Anderegg, P. A. M. Holweg, J. E. Mooij, H. Pothier, D. Esteve, C. Urbina and M. H. Devoret, Phys. Rev. Lett. **64**, 2691 (1990).
[3] L. DiCarlo, C. M. Marcus and J. S. Harris, Phys. Rev. Lett. **91**, 246804 (2003).
[4] M.G. Vavilov, L. DiCarlo, and C.M. Marcus, Phys. Rev. B **71**, 241309 (2005).
[5] P. J. Leek, M. R. Buitelaar, V. I. Talyanskii, C. G. Smith, D. Anderson, and G. A. C. Jones, J. Wei and D. H. Cobden, Phys. Rev. Lett. **95**, 256802 (2005).
[6] M. Büttiker, H. Thomas, and A. Prêtre, Z. Phys. B **94**, 133 (1994).
[7] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[8] I.L. Aleiner and A.V. Andreev, Phys. Rev. Lett. **81**, 1286 (1998).
[9] F. Zhou, B. Spivak, B. Altshuler: Phys. Rev. Lett. **82**, 608 (1999).
[10] J.E. Avron, A. Elgart, G.M. Graf, and L. Sadun, Phys. Rev. B **62**, R10618 (2000); Phys. Rev. Lett. **87**, 236601 (2001); J. Math. Phys. **43**, 3415 (2002).
[11] O. Entin-Wohlman, A. Aharony and Y. Levinson, Phys. Rev. B **65**, 195411 (2002).
[12] M. L. Polianski and P. W. Brouwer, Phys. Rev. B **64**, 075304 (2001).
[13] E. R. Mucciolo, C. Chamon, and C. M. Marcus Phys. Rev. Lett. **89**, 146802 (2002).
[14] J.E. Avron, A. Elgart, G.M. Graf, and L. Sadun, J. Stat. Phys. **116**, 425 (2004).
[15] H.-Q. Zhou, U. Lundin, S. Y. Cho and R. H. McKenzie, Phys. Rev. B **69**, 113308 (2004).
[16] M. Moskalets and M. Büttiker Phys. Rev. B **66**, 205320 (2002).
[17] M. Moskalets and M. Büttiker Phys. Rev. B **69**, 205316 (2004).
[18] M. Moskalets and M. Büttiker Phys. Rev. B **72**, 035324 (2005); cond-mat/0502106.
[19] S. Kohler, J. Lehmann, P. Hänggi, Phys. Rep. **406**, 379 (2005).
[20] L. Arrachea, Phys. Rev. B **72**, 125349 (2005); *ibid* **75**, 035319 (2007);
[21] L. Arrachea, Phys. Rev. B **72**, 121306 (2005); *ibid* **72**, 249904 (2005).
[22] L. Arrachea and M. Moskalets, Phys. Rev. B **74**, 245322 (2006)
[23] M.P. Anantram and F. Leonard, Rep. Progr. Phys. **69**, 507-561, (2006).
[24] K. Novoselov et al., Science **306**, 666, (2004); Nature **438**, 197, (2005).
Y. Zhang et al., Nature **438**, 201, (2005).
[25] C.M. Naón, M. J. Salvay, M.L. Trobo, Phys. Rev. B **70**, 195109, (2004).
C.M. Naón, M. J. Salvay, Phys. Rev. B **75**, 113102, (2007).
[26] S. Flach, O. Yevtushenko, Y. Zolotaryuk, Phys. Rev. Lett. **84**, 2358, (2000).
[27] G.D. Mahan, "Many Particle Physics", Plenum Press, Third Edition.
A. Das, "Finite Temperature Field Theory", World Scientific Pub. Co. Ltd. (1997).